

Power output analyses and optimizations of the Stirling cycle

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Based on the finite time thermodynamics theory, the entransy theory and the entropy theory, the Stirling cycles under different conditions are analyzed and optimized with the maximum output power as the target in this paper. The applicability of entransy loss (EL), entransy dissipation (ED), entropy generation (EG), entropy generation number (EGN) and modified entropy generation number (MEGN) to the system optimization is investigated. The results show that the maximum EL rate corresponds to the maximum power output of the cycle working under the infinite heat reservoirs whose temperatures are prescribed, while the minimum EG rate and the extremum ED rate do not. For the Stirling cycle working under the finite heat reservoirs provided by the hot and cold streams whose inlet temperatures and the heat capacity flow rates are prescribed, the maximum EL rate, the minimum EG rate, the minimum EGN and the minimum MEGN all correspond to the maximum power output, but the extremum ED rate does not. When the heat capacity flow rate of the hot stream increases, the power output, the EL rate, the EG rate and the ED rate increase monotonously, while the EGN and the MEGN decrease first and then increase. The EL has best consistency in the power output optimizations of the Stirling cycles discussed in this paper.

Stirling cycle, entropy generation, entransy loss, finite time thermodynamics, optimization

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1 Introduction

Under the background of the global energy shortage and the environment pollution, the exploration for the efficient and clean energy utilization becomes one of the focuses of the attention now. The Stirling engine, which is also called the external combustion hot-gas engine, has great potential for the future applications due to its high efficiency and good adaptive capacity for various kinds of energy and fuels [1]. Therefore, the optimization of the Stirling cycle is of great significance.

Curzon and Ahlborn [2] first proposed the concept of finite time thermodynamics (FTT) in 1975. As an extension of the traditional irreversible thermodynamics, the essence of the finite thermodynamics studies is to reduce the irre-

versibility and to optimize the thermal performances of practical systems by introducing heat transfer theory into thermodynamic analyses [3]. The FTT theory has been widely used by many researchers to analyze and optimize the energy systems, including heat engines, refrigerators, heat pumps, etc., under the constraints of finite heat transfer rate or finite size [4–25], promoting its rapid development [3, 26, 27]. As a typical model of the FTT analyses, the Stirling engine has received great attention. Li et al. [28] established the Stirling engine model with finite heat transfer rate, heat leak and finite regeneration time, and investigated the power and efficiency optimizations of the solar-powered Stirling heat engine by the FTT theory. They found that the absorber temperature and the regenerator effectiveness had significant effects on the performance of the Stirling engines. More FTT researches can be found in refs. [1, 29–34], etc.

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Entropy generation (EG) is usually regarded as the measure of the ability loss of doing work in thermodynamic cycles. Smaller EG indicates less loss of available energy. Therefore, the minimum EG is related to the optimal performance in thermodynamic cycle optimizations [35, 36]. Bejan [36] developed the EG minimization theory that takes the minimum EG as the optimization objective in considering the effect of finite temperature difference heat transfer on the total EG of the thermal system. Based on this theory, the minimum EG principle has been widely applied to the optimizations of thermal processes, such as heat-work conversion processes [11, 37–39]. However, arguments arose in its application. Some researches show that the minimum EG does not always correspond to the optimal performance of the thermal systems [40, 41]. The applicability of the minimum EG principle needs further discussion.

The entransy theory is newly developed in recent years and is applied to heat transfer optimizations. The concept of entransy is proposed by Guo et al. [42] from the analogy between the heat and electric conduction. Based on this concept, Guo et al. [42] developed the extremum ED principle and the minimum entransy-dissipation-based thermal resistance principle. These principles are used to optimize heat conduction [43, 44], heat convection [45–47], thermal radiation [48, 49] and the design of the heat exchangers [50, 51]. For the concept of entransy dissipation (ED), researches show that it is not appropriate for the optimization of heat-work conversion processes [52, 53]. However, efforts are made to extend the application of the entransy theory to the optimization of the heat-work conversion. Cheng et al. [41, 54] proposed the concept of entransy loss (EL), which is the sum of ED due to the irreversible heat transfer and the entransy variation due to the work output. EL is the entransy that is used in the heat-work conversion processes. The analyses of the air standard cycle [41] and the air conditioning system [54] show that the increase in the EL rate always leads to the increase in the power output. Compared with the optimization based on the concept of entropy, there is a better consistency between the variations of EL and the power output. However, there are not many researches related to EL, and no reports about the optimization of the thermodynamic cycles by combining FTT and EL currently.

Based on the FTT theory, the entransy and entropy theories are applied to the analyses of the Stirling cycles under different conditions in the present paper. The maximum power output is the objective. The applicability of the concepts of EL, ED, EG, etc., in the FTT optimization is discussed.

2 Model of the Stirling cycle

The typical simplified flow chart of the Stirling engine and its T - s diagram are shown in Figure 1. The Stirling engine is mainly composed of the compression space, the expansion

space and the regenerator, etc. There are four processes in the Stirling cycle as shown in Figure 1: (1) a-b, the isothermal process in which the working fluid absorbs heat and expands for doing work; (2) b-c, the isochoric process in which the working fluid keeps its volume and transfers heat to the regenerator; (3) c-d, the isothermal process in which the working fluid is compressed and releases heat to keep its temperature; (4) d-a, the isochoric process in which the working fluid keeps its volume and absorbs heat from the regenerator. For ideal regeneration, the heat absorbed during process d-a equals the heat released during process b-c. However, under the limits of finite regenerator size and the finite regenerative rate in practical applications, the ideal regeneration process cannot be realized. Therefore, the effect of the regenerator loss ΔQ_R is considered in the present paper.

For the Stirling cycle shown in Figure 1, assume that the working fluid is the ideal gas whose mass is n mol, and the working fluid temperatures at a-b and c-d processes are T_1 and T_2 , respectively. Then, the heat absorbed in process a-b and the heat released in process c-d by the working fluid are [35]

$$Q_1 = nRT_1 \ln \lambda, \quad (1)$$

$$Q_2 = nRT_2 \ln \lambda, \quad (2)$$

where R (J (mol K)⁻¹) is the universal gas constant, and $\lambda = V_b/V_a$ is the compression ratio. Assume that the effectiveness of the regenerator is η_m , the regenerated heat is [28]

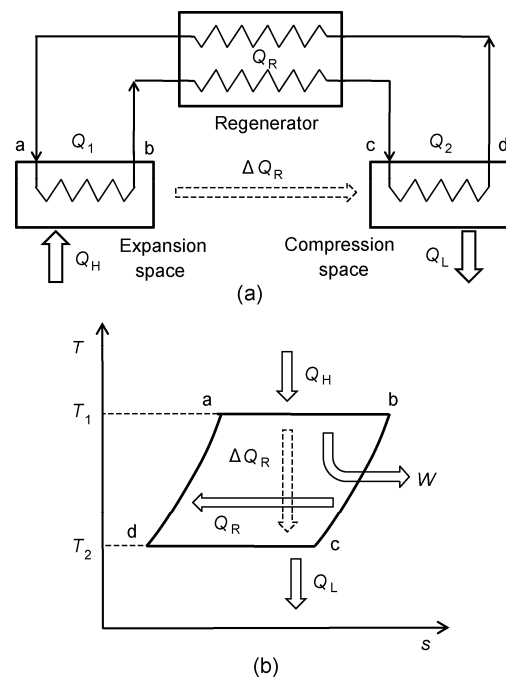


Figure 1 Flow chart and T - s diagram of the Stirling engine. (a) Typical simplified flow chart of the Stirling engine; (b) T - s diagram of the Stirling cycle.

$$Q_R = nc_v \eta_m (T_1 - T_2). \quad (3)$$

The regeneration loss due to the imperfection of the regeneration process is [28]

$$\Delta Q_R = nc_v (1 - \eta_m) (T_1 - T_2). \quad (4)$$

The existence of the regeneration loss is equivalent to a heat transfer of ΔQ_R that transfers directly from the hot working fluid to the cold working fluid. Therefore, the total heat supplied by the hot reservoir Q_H and that received by the cold reservoir Q_L in a cycle are

$$Q_H = Q_1 + \Delta Q_R = nRT_1 \ln \lambda + nc_v (1 - \eta_m) (T_1 - T_2), \quad (5)$$

$$Q_L = Q_2 + \Delta Q_R = nRT_2 \ln \lambda + nc_v (1 - \eta_m) (T_1 - T_2). \quad (6)$$

The following sections will show the analyses of the Stirling cycles with the maximum power output as the objective based on the FTT theory. We will discuss two different heat supplies, the infinite heat reservoir and finite heat reservoir, respectively.

3 FTT optimizations under heat sources of infinite heat reservoirs

As an external combustion engine, the Stirling engine can have various kinds of heat sources [35]. When the heat reservoirs are provided by vapor, etc., the heat reservoirs can be considered to have infinite heat capacities whose temperatures are constant. The T - s diagram of this cycle is shown in Figure 2.

Assume that the thermal conductances of the heat exchangers on the hot and cold sides are $(UA)_H$ and $(UA)_L$, respectively. The heat absorbed from the hot reservoir and that released to the cold reservoir by the working fluid during each cycle are

$$Q_H = (UA)_H (T_H - T_1) \tau_1, \quad (7)$$

$$Q_L = (UA)_L (T_2 - T_L) \tau_2, \quad (8)$$

where τ_1 and τ_2 are the times of process a-b and process c-d,

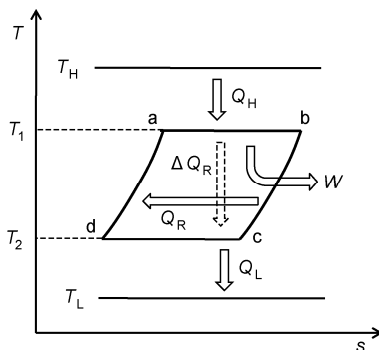


Figure 2 T - s diagram of the Stirling cycle under infinite heat reservoirs.

respectively. The combination of eqs. (5)–(8) gives

$$\tau_1 = \frac{nRT_1 \ln \lambda + nc_v (1 - \eta_m) (T_1 - T_2)}{(UA)_H (T_H - T_1)}, \quad (9)$$

$$\tau_2 = \frac{nRT_2 \ln \lambda + nc_v (1 - \eta_m) (T_1 - T_2)}{(UA)_L (T_2 - T_L)}. \quad (10)$$

Considering that the regeneration processes are executed with finite heat transfer rate and assuming that the temperature of the working fluid varies at a constant rate, the times of the regeneration processes b-c and a-d can be expressed as [28]

$$\tau_3 = (T_1 - T_2) / M_1, \quad (11)$$

$$\tau_4 = (T_1 - T_2) / M_2, \quad (12)$$

where M_1 and M_2 are the constants related to the properties of the regenerator such as materials, structures, etc.

Based on the analyses above, the cycle time of the Stirling cycle can be obtained

$$\tau = \tau_1 + \tau_2 + \tau_3 + \tau_4, \quad (13)$$

and the cycle output power is

$$\dot{W} = (Q_H - Q_L) / \tau = nR \ln \lambda (T_1 - T_2) / \tau. \quad (14)$$

According to its definition, EL is the difference between the entransy flows that get into and out of the system [54]. Dividing EL of each cycle by the cycle time, the EL rate of the cycle is

$$\dot{G}_{\text{loss}} = (Q_H T_H - Q_L T_L) / \tau. \quad (15)$$

EG and ED in a cycle are the respective sums of the EG and ED in all the processes. Therefore, the EG rate and the ED rate of the engine are

$$\begin{aligned} \dot{S}_{\text{gen}} &= \left(\frac{Q_H}{T_1} - \frac{Q_H}{T_H} + \frac{Q_L}{T_L} - \frac{Q_L}{T_2} + \frac{\Delta Q_R}{T_2} - \frac{\Delta Q_R}{T_1} \right) / \tau \\ &= (Q_L / T_L - Q_H / T_H) / \tau, \end{aligned} \quad (16)$$

$$\begin{aligned} \dot{G}_{\text{dis}} &= [Q_H (T_H - T_1) + Q_L (T_2 - T_L) + \Delta Q_R (T_1 - T_2)] / \tau \\ &= (Q_H T_H - Q_L T_L - Q_1 T_1 + Q_2 T_2) / \tau. \end{aligned} \quad (17)$$

It can be seen from eqs. (5)–(14) that the variations of the working temperatures T_1 and T_2 will affect the output work and the cycle time at the same time when the temperatures of the hot and cold streams T_H and T_L are prescribed, and will consequently lead to the variation of the output power. For the objective of the maximum power output, the working temperatures T_1 and T_2 need to be optimized.

Assume that the working temperatures T_1 and T_2 and the heat reservoir temperatures T_H and T_L satisfy [34]

$$T_2 / T_1 = (T_L / T_H)^r, \quad (18)$$

where r is the isochoric temperature ratio exponent. The objective of the present problem is to find the optimal T_1 that leads to the maximum power output under arbitrarily prescribed r , and to investigate the applicability of the entransy and entropy theories to the optimization.

Let us consider a numerical example. Assume that $r = 0.6$, $T_H = 1000$ K, $T_L = 300$ K, $(UA)_H = 300$ W K⁻¹, $(UA)_L = 300$ W K⁻¹, $\eta_m = 0.8$, $c_v = 20$ J (mol K)⁻¹, $R = 8.314$ J (mol K)⁻¹, $\lambda = 3$, $n = 0.5$ mol and $M_1 = M_2 = 8 \times 10^4$ K s⁻¹. The practical parameters are different for various engines due to the difference in the structures, working fluids, etc. As the present work discusses the applicability of the entransy and entropy to the optimization of the Stirling cycles, the above parameters are not originated from a real Stirling engine, but are selected in the general range of the practical engine parameters, and are similar to values in refs. [28, 31].

The output power, the EL rate, the EG rate and the ED rate with different working temperature T_1 are calculated based on the above parameters, and are shown in Figure 3. It can be seen that all the parameters increase first and then decrease with increasing T_1 . The output power, the EL rate and the EG rate reach their maximum values when $T_1 = 817$ K. Therefore, the maximum EL rate corresponds to the maximum output power for this problem. The minimum EG rate does not correspond to the maximum output power, but the maximum EG rate does. The ED rate reaches its maximum value at $T_1 = 782$ K, which means that the extremum ED rate does not correspond to the maximum output power. Thus, the ED rate is not suitable for the output power optimization.

The above results are analyzed below. The expression of the EL rate for this cycle can be changed into

$$\begin{aligned} \dot{G}_{\text{loss}} &= \frac{Q_H T_H - Q_L T_L}{\tau} = \frac{W(T_H - T_L) / \eta + W T_L}{\tau} \\ &= \frac{\dot{W} [T_H - (1 - \eta) T_L]}{\eta}, \end{aligned} \quad (19)$$

where η is the cycle efficiency. Similarly, the expressions of the EG rate and the ED rate also can be changed into

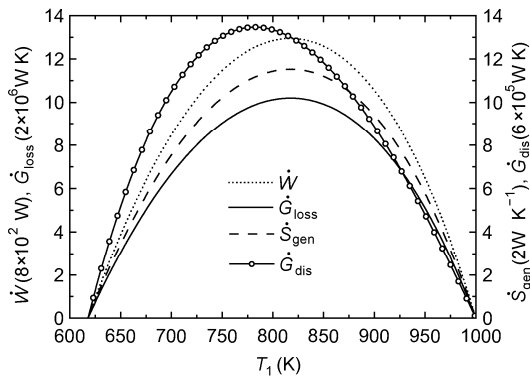


Figure 3 Variations of the output power, the EL rate, the EG rate and the ED rate with T_1 .

$$\dot{S}_{\text{gen}} = \frac{\dot{W} [(1 - \eta) T_H - T_L]}{\eta T_H T_L}, \quad (20)$$

$$\dot{G}_{\text{dis}} = \frac{T_H - (1 - \eta) T_L + \eta T_2}{\eta} \dot{W} + \frac{Q_1 (T_1 - T_2)}{\tau}. \quad (21)$$

For the Stirling cycle working under infinite heat reservoirs, the cycle efficiency can be calculated based on eqs. (5) and (6),

$$\begin{aligned} \eta &= (Q_H - Q_L) / Q_H \\ &= \frac{n R T_1 \ln \lambda - n R T_2 \ln \lambda}{n R T_1 \ln \lambda + n c_v (1 - \eta_m) (T_1 - T_2)} \\ &= \frac{1 - T_2 / T_1}{1 + [c_v (1 - \eta_m) (1 - T_2 / T_1)] / (R \ln \lambda)}. \end{aligned} \quad (22)$$

This equation and eq. (18) show that the cycle efficiency is constant when T_H , T_L and r are prescribed. Eqs. (19)–(21) show that the EL rate and the EG rate increase monotonously with increasing output power when the cycle efficiency is prescribed, while the ED rate has no direct relation to the output power under the same condition.

From the viewpoint of entransy, there are three necessary processes in heat-work conversion, and the corresponding costs must be paid for each process. Firstly, heat is transferred from the hot reservoir to the working fluid, and the corresponding cost is ED. Secondly, heat is converted to work by the thermodynamic cycle. Heat entransy is converted to work entransy in this process that leads to the decrease of heat entransy. Finally, the used heat is transferred to the cold reservoir, and the cost of this process is also the corresponding ED. Overall, EL is the necessary costs of the systems for outputting work. Therefore, the EL rate can be applied to the optimization of the output power. However, ED does not describe all the processes of the heat-work conversion. That is why the ED rate cannot be used for the output power optimization.

Bejan [36] introduced the concept of entropy generation number (EGN) when EG is used for the optimization of the thermal processes involving heat exchangers. EGN is expressed as

$$N_s = \dot{S}_{\text{gen}} / C_{\text{min}}, \quad (23)$$

where C_{min} is the minimum heat capacity flow through the heat exchangers. The “EG paradox” is noted when EGN is used in the analyses of the counter flow heat exchangers [36]. To remove the paradox, Xu and Yang [55] defined the modified entropy generation number (MEGN), which is the production of the EG per unit heat transfer rate and the ambient temperature. The expression is

$$N_{\text{gs}} = \dot{S}_{\text{gen}} T_0 / \dot{Q}. \quad (24)$$

For the present problem, EGN cannot be defined because there is no heat capacity flows through the heat exchangers.

For MEGN, combining eqs. (20) and (24) yields,

$$N_{gs} = T_0 [(1-\eta)T_H - T_L] / (T_H T_L). \quad (25)$$

Eq. (25) shows that MEGN is a constant when T_H , T_L , T_0 and r are prescribed. The output power has an extremum value with the variation of the working temperature T_1 in such condition. So, MEGN cannot be used for the output power optimization in such prescribed conditions.

The case in which $r = 0.6$ is discussed above. Further analysis of the above equations shows that the relationship remains the same for arbitrary values of r . The maximum EL rate and the maximum EG rate always correspond to the maximum output power with infinite heat reservoirs of constant temperature, while the ED rate and the MEGN have no direct relation with the output power.

4 FTT optimizations under heat sources of finite heat reservoirs

The Stirling engine has great potential for the applications in the field of the waste heat recovery due to its high efficiency [35]. The used water, gas, etc., at certain temperature are the common waste heat sources in industry, and the heat can be transferred to the Stirling engines through heat exchangers to produce power output. The flow chart and T - s diagram in such an application are shown in Figure 4. The hot stream at temperature T_{H-in} flows through the exchanger on hot side with heat capacity flow rate C_H , and the cold stream at temperature T_{L-in} flows through the exchanger on cold side with heat capacity flow rate C_L . The used streams

are dumped into the environment at temperature T_0 .

For such an engine, the heat absorbed from the hot stream and that released to the cold stream by the working fluid during a cycle are

$$Q_H = C_H (T_{H-in} - T_{H-out}) \tau_1 = \varepsilon_H C_{H-min} (T_{H-in} - T_1) \tau_1, \quad (26)$$

$$Q_L = C_L (T_{L-out} - T_{L-in}) \tau_2 = \varepsilon_L C_{L-min} (T_2 - T_{L-in}) \tau_2, \quad (27)$$

where T_{H-out} and the T_{L-out} are the outlet temperatures of the hot and cold streams, respectively, and C_{H-min} and C_{L-min} are

$$C_{H-min} = \min\{C_H, C_g\}, \quad C_{L-min} = \min\{C_L, C_g\}. \quad (28)$$

where C_H , C_L and C_g are the heat capacity flow rates of the hot stream, the cold stream and the working fluid through the heat exchangers, respectively. As the working fluid experiences the isothermal processes in the exchangers, C_g can be considered as infinity. Therefore, $C_{H-min} = C_H$ and $C_{L-min} = C_L$. The effectivenesses of the exchangers on the hot and cold sides, ε_H and ε_L , can be calculated by

$$\varepsilon_H = 1 - \exp[-(UA)_H / C_{H-min}], \quad (29)$$

$$\varepsilon_L = 1 - \exp[-(UA)_L / C_{L-min}], \quad (30)$$

where $(UA)_H$ and $(UA)_L$ are the thermal conductance of the heat exchangers on hot and cold sides, respectively.

Combining eqs. (5), (6), (26) and (27), the times of process a-b and process c-d can be calculated,

$$\tau_1 = \frac{nRT_1 \ln \lambda + nc_v(1-\eta_m)(T_1 - T_2)}{\varepsilon_H C_{H-min} (T_{H-in} - T_1)}, \quad (31)$$

$$\tau_2 = \frac{nRT_2 \ln \lambda + nc_v(1-\eta_m)(T_1 - T_2)}{\varepsilon_L C_{L-min} (T_2 - T_{L-in})}. \quad (32)$$

The times of regeneration processes a-d and b-c, τ_3 and τ_4 , can also be obtained by eqs. (11) and (12), and then the cycle time can be obtained by eq. (13).

We use a similar isochoric temperature ratio exponent r with which the working temperatures T_1 and T_2 and the inlet temperatures T_{H-in} and T_{L-in} satisfy

$$T_2 / T_1 = (T_{L-in} / T_{H-in})^r. \quad (33)$$

According to the analyses above, for arbitrary r , all the parameters of the system can be obtained based on eqs. (5), (6), and (26)–(33) with different T_1 . Then, the output power of the cycle is

$$\dot{W} = (Q_H - Q_L) / \tau = [nR \ln \lambda (T_1 - T_2)] / \tau. \quad (34)$$

The entransy and entropy analyses are carried out below. Dividing the EL of each cycle by the cycle time, the EL rate can be obtained as

$$\begin{aligned} \dot{G}_{loss} = & \frac{1}{2} C_H (T_{H-in}^2 - T_0^2) + \frac{1}{2} C_L (T_{L-in}^2 - T_0^2) \\ & - [C_H (T_{H-out} - T_0) \tau_1 + C_H (T_{H-in} - T_0) (\tau - \tau_1) \\ & + C_L (T_{L-out} - T_0) \tau_2 + C_L (T_{L-in} - T_0) (\tau - \tau_2)] \frac{T_0}{\tau}, \end{aligned} \quad (35)$$

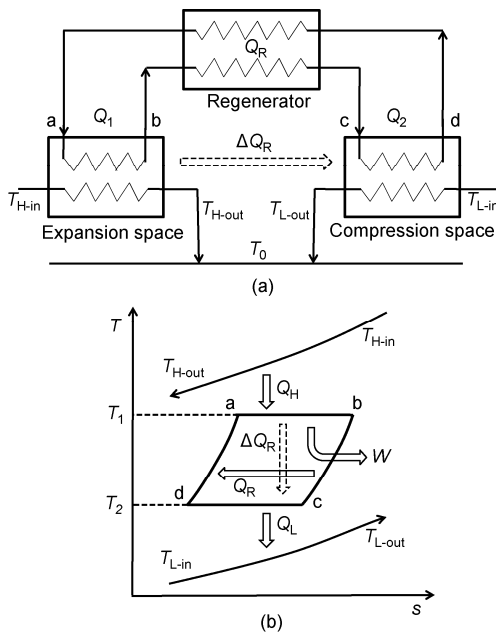


Figure 4 Flow chart and T - s diagram of the Stirling engine under finite heat reservoirs. (a) Simplified flow chart of the Stirling cycle under finite heat reservoirs; (b) T - s diagram of the Stirling cycle under finite heat reservoirs.

where the EL induced by dumping the used streams into the environment is considered [54]. The first two terms on the right hand side are the decrements of the entransy flow rate of the hot and cold streams from the inlet to the environment, and the third term is the increment of entransy flow rate of the environment. In one cycle time interval, τ , the heat transfer between the hot stream and the working fluid only takes τ_1 , the heat transfer between the cold stream and the working fluid only takes τ_2 , and the streams dump to the environment directly in the rest time. Eq. (35) can be simplified into

$$\begin{aligned}\dot{G}_{\text{loss}} &= \frac{1}{2}C_H(T_{H-\text{in}}^2 - T_0^2) + \frac{1}{2}C_L(T_{L-\text{in}}^2 - T_0^2) \\ &\quad - C_H(T_{H-\text{in}} - T_0)T_0 - C_L(T_{L-\text{in}} - T_0)T_0 \\ &\quad + \frac{[C_H(T_{H-\text{in}} - T_{H-\text{out}})\tau_1 + C_L(T_{L-\text{in}} - T_{L-\text{out}})\tau_2]T_0}{\tau} \\ &= \frac{1}{2}C_H(T_{H-\text{in}} - T_0)^2 + \frac{1}{2}C_L(T_{L-\text{in}} - T_0)^2 + \dot{W}T_0.\end{aligned}\quad (36)$$

It can be seen that the EL rate would increase monotonously with increasing output power when the heat capacity flow rates, the stream inlet temperatures and the ambient temperature are prescribed.

Dividing the total EG of each cycle by the cycle time, the EG rate is

$$\begin{aligned}\dot{S}_{\text{gen}} &= C_H \ln \frac{T_0}{T_{H-\text{in}}} + C_L \ln \frac{T_0}{T_{L-\text{in}}} \\ &\quad + [C_H(T_{H-\text{out}} - T_0)\tau_1 + C_H(T_{H-\text{in}} - T_0)(\tau - \tau_1) \\ &\quad + C_L(T_{L-\text{out}} - T_0)\tau_2 + C_L(T_{L-\text{in}} - T_0)(\tau - \tau_2)]/(\tau T_0),\end{aligned}\quad (37)$$

where EG induced by dumping the used streams into the environment is considered [36]. The first two terms on the right hand side are the entropy variation rates of the hot and cold streams from the inlet to the environment, respectively, and the third term is the entropy variation rate of the environment. Eq. (37) can be simplified into

$$\begin{aligned}\dot{S}_{\text{gen}} &= C_H \ln \frac{T_0}{T_{H-\text{in}}} + C_L \ln \frac{T_0}{T_{L-\text{in}}} \\ &\quad + \frac{C_H(T_{H-\text{in}} - T_0) + C_L(T_{L-\text{in}} - T_0)}{T_0} \\ &\quad + \frac{C_H(T_{H-\text{out}} - T_{H-\text{in}})\tau_1 + C_L(T_{L-\text{out}} - T_{L-\text{in}})\tau_2}{\tau T_0} \\ &= C_H \ln \frac{T_0}{T_{H-\text{in}}} + C_L \ln \frac{T_0}{T_{L-\text{in}}} \\ &\quad + \frac{C_H(T_{H-\text{in}} - T_0) + C_L(T_{L-\text{in}} - T_0)}{T_0} - \frac{\dot{W}}{T_0}.\end{aligned}\quad (38)$$

It can be seen that EG also decreases monotonously with increasing output power when the heat capacity flow rates, the inlet temperatures of the streams and the ambient tem-

perature are prescribed.

Similarly, dividing the total ED of a cycle by the cycle time, the ED rate is

$$\begin{aligned}\dot{G}_{\text{dis}} &= \frac{1}{2}C_H(T_{H-\text{in}}^2 - T_0^2) + \frac{1}{2}C_L(T_{L-\text{in}}^2 - T_0^2) \\ &\quad - [(Q_1T_1 - Q_2T_2)/\tau] \\ &\quad - [C_H(T_{H-\text{out}} - T_0)\tau_1 + C_H(T_{H-\text{in}} - T_0)(\tau - \tau_1) \\ &\quad + C_L(T_{L-\text{out}} - T_0)\tau_2 + C_L(T_{L-\text{in}} - T_0)(\tau - \tau_2)]\frac{T_0}{\tau},\end{aligned}\quad (39)$$

where ED induced by dumping the used streams into the environment is also considered. Eq. (39) can also be simplified into

$$\begin{aligned}\dot{G}_{\text{dis}} &= \frac{1}{2}C_H(T_{H-\text{in}}^2 - T_0^2) + \frac{1}{2}C_L(T_{L-\text{in}}^2 - T_0^2) \\ &\quad - C_H(T_{H-\text{in}} - T_0)T_0 - C_L(T_{L-\text{in}} - T_0)T_0 \\ &\quad - [(Q_1T_1 - Q_2T_2)/\tau] \\ &\quad + \{[C_H(T_{H-\text{in}} - T_{H-\text{out}})\tau_1 + C_L(T_{L-\text{in}} - T_{L-\text{out}})\tau_2]T_0\}/\tau \\ &= \frac{1}{2}C_H(T_{H-\text{in}} - T_0)^2 + \frac{1}{2}C_L(T_{L-\text{in}} - T_0)^2 \\ &\quad - [(Q_1T_1 - Q_2T_2)/\tau] + \dot{W}T_0.\end{aligned}\quad (40)$$

There is no specific relationship between the variations of the ED rate and the output power even when all the inlet parameters of the streams are prescribed.

Numerical examples are given in the following paragraph to illustrate the applicability of EL, EG and ED to the optimization of the system with the objective of the maximum power output.

The case in which T_1 is optimized under the prescribed r is discussed first. Assume that $r = 0.6$, $T_{H-\text{in}} = 800$ K, $T_{L-\text{in}} = 290$ K, $T_0 = 300$ K, $C_H = 40$ W K⁻¹, $C_L = 100$ W K⁻¹, $(UA)_H = 50$ W K⁻¹, $(UA)_L = 50$ W K⁻¹, $\eta_m = 0.8$, $c_v = 20$ J (mol K)⁻¹, $R = 8.314$ J (mol K)⁻¹, $\lambda = 3$, $n = 0.5$ mol and $M_1 = M_2 = 8 \times 10^4$ K s⁻¹. According to the above parameters and equations, the output power, the EL rate, the EG rate and the ED rate with different T_1 are calculated as shown in Figure 5. It can be seen that the output power and the EL rate both increase first and then decrease with increasing T_1 , while the EG rate and the ED rate both decrease first and then increase. When $T_1 = 660$ K, the output power and the EL rate reach their maximum values, and the EG rate reaches its minimum value. However, the ED rate reaches its minimum value when $T_1 = 679$ K. Thus, the maximum EL rate and the minimum EG rate both correspond to the maximum output power for the present case, while the extremum ED rate does not. Therefore, the maximum EL rate and the minimum EG rate both can be used in the optimization of this case, which is unanimous with the analyses above.

For arbitrary r , the optimal working temperature T_1 that leads to the maximum output power, the maximum EL rate and the minimum EG rate can be found. Therefore, a case

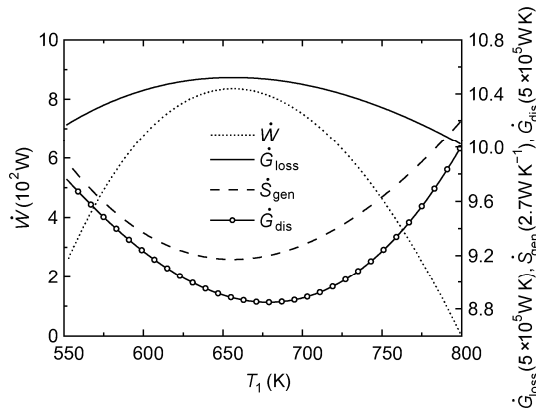


Figure 5 Variations of the output power, the EL rate, the EG rate and the ED rate with T_1 .

with the variation of r is discussed next. For each r , the maximum output power, the maximum EL rate and the minimum EG rate at the optimal T_1 are calculated and shown in Figure 6. The maximum output power and the maximum EL rate both increase first and then decrease with increasing r , while the minimum EG rate decreases first and then increases. When $r = 0.45$, the output power and the EL rate reach their maximum values, and the EG rate reaches its minimum value. Therefore, the maximum EL rate and the minimum EG rate both can be used for the optimization of r to obtain the maximum output power.

Moreover, the EGN of this problem can be calculated by eq. (23), where $C_{\min} = \min\{C_H, C_g, C_L\}$. According to eq. (24), the MEGN is

$$N_{gs} = \dot{S}_{gen} T_0 / [C_H (T_{H-in} - T_0) + C_L (T_{L-in} - T_0)]. \quad (41)$$

Eqs. (23) and (41) reveal that the EGN and the MEGN both have the same variation tendencies with the EG rate when the inlet parameters (temperature, heat capacity flow) of the streams and the ambient temperature are prescribed, which demonstrates that both the EGN and the MEGN can be used for the power output optimization under the conditions above.

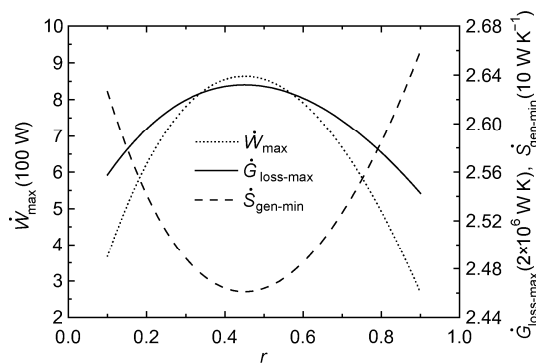


Figure 6 Variations of the maximum output power, the EL rate and the minimum EG rate with r .

Another case in which the heat capacity flow rate of the hot stream C_H is not prescribed is considered. In this case, the total heat flow rate into the system is not constant. The working temperature is fixed at $T_1 = 656$ K and other parameters are the same as the above numerical example. The output power, the EL rate, the EG rate and the ED rate are calculated and shown in Figure 7. All the parameters in Figure 7 increase monotonously with increasing C_H , which means that larger EL rate and larger ED rate both correspond to larger output power, while smaller EG rate does not. Therefore, the maximum EL rate and the extremum ED rate are suitable for the optimization of this case, but the minimum EG rate is not. Furthermore, the variations of EGN and MEGN with C_H are shown in Figure 8. It can be seen that EGN and MEGN both decrease first and then increase with increasing C_H . As the output power increases monotonously with increasing C_H , neither of EGN and MEGN fits the output power optimization when C_H is not prescribed.

Actually, EG is a physical quantity that measures the irreversibility of processes. EG can be affected by two important factors in heat-work conversion. One is the temperature difference of the heat transfer, and the other is the total

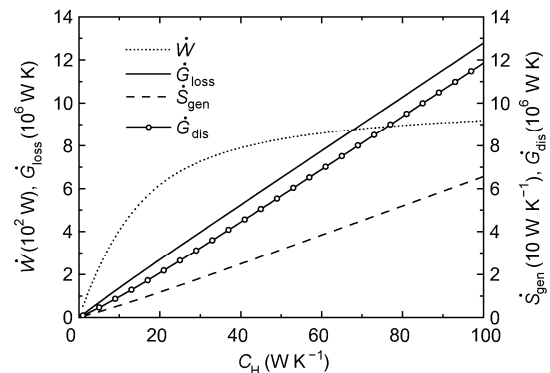


Figure 7 Variations of the output power, the EL rate, the EG rate and the ED rate with C_H .

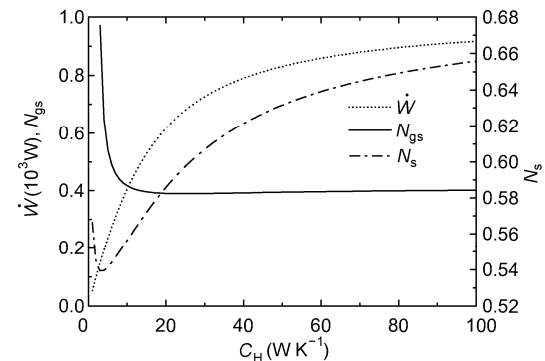


Figure 8 Variations of the output power, the EGN and the MEGN with C_H .

heat transferred. When the inlet parameters of the hot streams are prescribed, the total heat that can be used for doing work is prescribed. Under this condition, smaller EG rate means less loss of the ability of doing work, which leads to larger output power. When the inlet parameters are not prescribed, the total heat that can be used for doing work is not prescribed. The output power of the system is not only related to the temperature difference of the heat transfer, but also to the total heat input. With the increase in the total heat input, the output power increases, and the EG rate increases too. Consequently, the minimum EG rate is not a suitable criterion for the output power optimization in such a condition. Overall, EG does not describe the output power of the system directly, but indirectly by the loss of the ability of doing work. Just due to this indirectness, limitations are encountered when EG is used for optimizing the thermal processes.

5 Conclusions

The entransy theory is combined with the finite time thermodynamic theory to analyze the Stirling engine cycles in this paper. The applicability of the entransy and entropy theories to the optimizations of the cycle with the objective of the maximum output power is investigated.

(1) For the cycle working under the infinite heat reservoirs whose temperatures are prescribed, the maximum EL rate can be used for optimizing the power output. The minimum EG rate is not related to the maximum power output; instead, the maximum EG rate corresponds to the maximum output power. The extremum ED rate has no direct relation to the power output and is not appropriate for the optimization. EGN cannot be defined, and MEGN also has no direct relation to the output power because it is a constant when r is prescribed.

(2) For the cycle working under the finite heat reservoirs provided by the streams, the maximum EL rate, the minimum EG rate, the minimum EGN and the minimum MEGN all can be used for the output power optimization when the stream inlet temperatures and the heat capacity flow rates are prescribed except the extremum ED rate. When the heat capacity flow of the hot stream is not prescribed, the minimum EG rate does not corresponds to the maximum output power, but the maximum EL still does.

(3) The concept of entransy loss considers the effects of the heat transfer processes and the work doing processes on the variation of the entransy, i.e., ED and work entransy. For the cases discussed in this paper, the increase in heat transfer rate leads to the increase in output power and the increase in the ED rate, and finally results in the increase in the EL rate. Similarly, the increase of the output work leads to the increase in work entransy flow rate, and then the EL rate also increases. Thus, a better consistency is discovered when the concept of EL is used for the output power opti-

mization of the Stirling cycle. For the concept of EG, larger output power corresponds to the smaller EG rate when the total heat input from the hot stream is prescribed. When the total heat input from the hot stream increases under the prescribed temperature, the output power increases and the EG rate increases, too. The relationship between the variations of the output power and the EG rate is affected by the preconditions, which makes the applicability of EG, EGN and MEGN conditional. The concept of ED is not always appropriate for the heat-work conversion optimization because it does not consider the effect of the work doing processes on the variation of the entransy.

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